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Complex Networks Approach for Studying Polarization in Different Social Groups

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Abstract. We study the emergence of polarization in a social group characterized by the specific network topology. We propose a discrete-time model to describe the individuals' opinion state evolution. We consider the size and the average degree of each network for comparing how these variables influence the polarization. To complete our description of how the polarization occurs, we introduce the concept of susceptible individuals becoming zealots and intransigents that allows us to obtain an index to measure such a polarization index as a function of the susceptible individuals' fraction present at the beginning of each numerical experiment. With all these elements, we determine how the social group is polarized. We emphasize the susceptible individuals' fraction's essential role in triggering the polarization. On the other hand, it is interesting to point out that, in general, the polarization decreases as the number of nodes and the average degree increase except small-world networks, which enhance the appearance of polarization.

INTRODUCTION

The study of social systems from a physical approach has become an important topic in complex systems [1] that sometimes is also called Sociophysics [2]. The individuals' opinion constitutes the primary variable, and its evolution features a social system. Nowadays, there is a boom in the study of social systems, due on the one hand, to the fact that many social networks have arisen in the last years, involving millions of individuals. On the other hand, the availability of a vast quantity of data and the development of new tools to treat and analyze such an enormous quantity of data on specific social systems. Moreover, the formulation of models describing social systems is also prolific; and the contrast of model results with real data allows for a convenable description of the social opinion dynamics [3, 4]. Opinion dynamics is important because it permits the analysis of factors that control or influence it, such as the network topology [5, 6, 7], as well as the introduction of intransigent individuals [8, 9] or zealots [10] who can drive the evolution of opinion towards phenomena such as consensus or polarization of society, which are phenomena of vital importance. After all, they can lead to a series of unwanted effects on democratic institutions and can lead to biased decision-making. Furthermore, in a polarized society, false information can spread quickly, leading to increased intolerance of opposing views and segregation of ideologies. Due to the polarization of opinions, moderate positions lose their influence, leading to an unstable society exacerbated by convulsion and violence.

In this work, we focus on the emergence of polarization as a function of the network features and the initial conditions related to the susceptible individuals to turn out on zealots. The article is structured as follows: Firstly, we introduce the used model to explain zealots' appearance in populations with different susceptible individual fractions. Simultaneously, we propose an index to characterize the polarization, which constitutes the primary variable to express the results. Finally, we state the conclusions and perspectives.

THE MODEL

We adopt a discrete-time model for the dynamical description of a social group composed of N individuals. This model considers the opinion state of each member j, $S_j(t)$ at a time $t+1$, which depends on its previous value $S_j(t)$ and it is also influenced by the opinion of their connections, who might be acquaintances or friends of j. The described social group constitutes an undirected and unweighted network characterized by its adjacency matrix, represented by the matrix elements a_{ij}, which take the values one or zero according to whether or not a link exists between the

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individuals [11]. Thus, the resulting equation reads as:

$$S_j(t+1) = \frac{S_j(t) + \sum_{i=1}^{N} a_{ij} S_i(t)}{1 + \sum_{i=1}^{N} a_{ij}} \ ; \text{ with } j = 1,2,...,N \ . \tag{1}$$

Note that in this model, the opinion state value of the individual j depends on its initial condition $S_j(0)$. Initially, two individuals are assigned as seeds, with extreme and constant opinion values ($S(0) = 0$ or 1). Then, on the other hand, the remaining individuals' opinions are randomly chosen from a Gaussian distribution with a mean of 0.5. The resulting values of the opinion state of each individual can take continuous values between 0 and 1. Consequently, we consider that when an opinion state exceeds the upper and lower thresholds, i.e., above 0.7 or below 0.3; thus, the opinion state is related to an extreme opinion in a positive or negative sense. After a finite number of time steps, it is observed that the individuals' opinion attains a stable and constant value. Therefore, the system as a whole stabilizes as well.

We define a susceptible individual as one whose initial neutral opinion evolves until it exceeds one of the threshold values stated above, either the positive (0.7) or negative (0.3). Once it occurs, the individual becomes a zealot and intransigent, immediately acquiring an extreme opinion of 1 or 0. As the individual adopts an intransigent and radical position, this extreme opinion does not change anymore despite the received influences from the other members of the network.

Our primary concern lies in the opinion polarization in a social network that is understood as the network split into two predominant groups characterized by their opinion states whose values are extremes but contraries. The fractions of individuals whose opinion states are above 0.7 and below 0.3 are denoted by fp and fn, respectively. These quantities are related to the polarization of the social system. Nevertheless, note that it is not enough that individuals reach extreme opinions to classify the system as polarized. For instance, if all the individuals of a social system attain the same extreme opinion state, whether fp or fn, then the system is considered to achieve consensus, a behavior opposed to polarization. A fully polarized state shall have half of the individuals with an extremely positive or negative opinion, i.e., $fp = fn = 50\%$. This situation would describe a fully polarized system. A social network with dispersed but not extreme opinions would not be polarized either. Considering the aspects mentioned above, we introduce the following polarization index:

$$R_{\%} = [1 - 2\,(0.5 - \min\{fp, fn\})] \times 100 \ . \tag{2}$$

The values of R are in percentage, where a value of zero indicates no polarization and, on the contrary, $R = 100\%$ states for the maximum polarization.

RESULTS

In Fig. 1, we show the results of polarization as a function of the fraction of susceptible individuals obtained in a hundred experiments under the same characteristics. For each experiment it has been varied the number of the individuals N, the average degree of the network $\langle k \rangle$ (characterized by the color, magenta represent the smallest $\langle k \rangle$ value and cyan the biggest $\langle k \rangle$ value). The experiments were made for three types of networks: random, scale-free and small-world networks that describe the relation between individuals.

As expected, in all types of networks, it is observed that polarization grows with the fraction of susceptible individuals in the system. However, in the scale-free network, when there is a quite small average degree $\langle k \rangle = 1$ (magenta curves) as in Figs. 1(b),(e), and (h). This fact is not evident because, with such a low number of links for this type of network, the initial opinion (assigned randomly) generates a significant polarization value, remaining in time. To imagine this, we must remember that just a few nodes have contact with many others in the scale-free network. Therefore, it is unlikely that these particular nodes have extreme opinions. It is necessary for a more significant interaction between individuals for an evolution to exist.

If the fraction of susceptible is zero, the polarization tends to be zero. Except when the average degree $\langle k \rangle$ is relatively small, that does not allow the evolution explained above. In all cases, even if there are 100% susceptible individuals, the polarization does not reach 100%. For random and scale-free networks (Figs. 1(a), (d) and (g) and

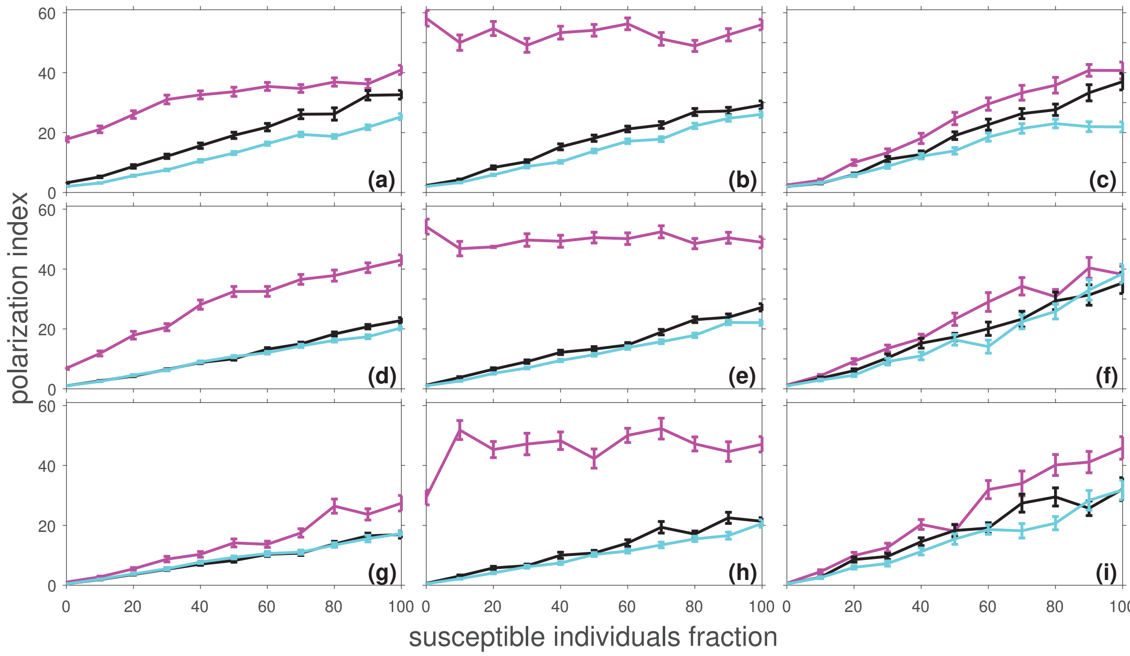

FIGURE 1. Polarization vs. the susceptible individuals' fraction. Each column of figures was obtained for a different network: Figures (a), (d) and (g) for random networks, figures (b), (e) and (h) for scale-free networks and figures (c), (f) and (i) for small-world networks. Each row of figures was obtained for a different number of individuals N: Figures (a), (b) and (c) for $N = 100$, figures (d), (e) and (f) for $N = 200$ and figures (g), (h) and (i) for $N = 400$. The curve color is related to the average degree $\langle k \rangle$, magenta is the shortest $\langle k \rangle$ value and cyan is the largest $\langle k \rangle$ value.

Figs. 1(b), (e) and (h) respectively), we observe that as N increases, the polarization obtained decreases, but this is not so evident in small-world networks (Figs. 1(c), (f) and (i)).

On the other hand, in the three types of studied networks, when we consider the same N, it is observed that the curves are ordered as follows: magenta, black, and cyan. Consequently, as $\langle k \rangle$ increases, the polarization decreases. However, note that when N takes a significant value Figs. 1(g) and (h), the cyan and black lines almost overlap; that is, the degree is no more relevant when N is big. In Fig. 2 we show a comparison of the polarization index as a function of susceptible individuals fraction, obtained for different types of networks and average degrees $\langle k \rangle$, but with the same number of individuals N. Random, scale-free, and small-world networks are represented respectively by black, red, and yellow lines for the same number of nodes $N = 200$ and different average degrees $\langle k \rangle$. When $\langle k \rangle \neq 1$, it is observed that the polarization index R_{SW} related to the small-world networks is greater than the polarization indices related to scale-free R_{SF} and random R_R networks. Nevertheless, this happens when the susceptible individuals' fractions are greater than around 40% as shown in Figs. 2(b)–(c). When $\langle k \rangle = 1$, it has been already explained that due to the average degree smallness, the opinion state evolution does not significantly change in the case of the scale-free network as shown in Fig. 2(a). In Fig. 3 we compare the polarization indices obtained for different types of networks (random, scale-free, and small-world) and number of nodes ($N = 100$, $N = 200$, and $N = 400$), but with the same average degree, $\langle k \rangle = 3$. We observe that as N increases, R_{SF} falls dramatically (see especially Fig. 3(c); on the other hand, R_{SW} remains almost constant for different numbers of nodes N.

CONCLUSIONS AND PERSPECTIVES

This work describes the mechanism of attaining polarization in a social group through a simple discrete-time model depicting the opinion state evolution. Furthermore, we considered that the group's social structure is related to specific network topology (random, scale-free, or small-world). Therefore, we vary the size of the group and the average

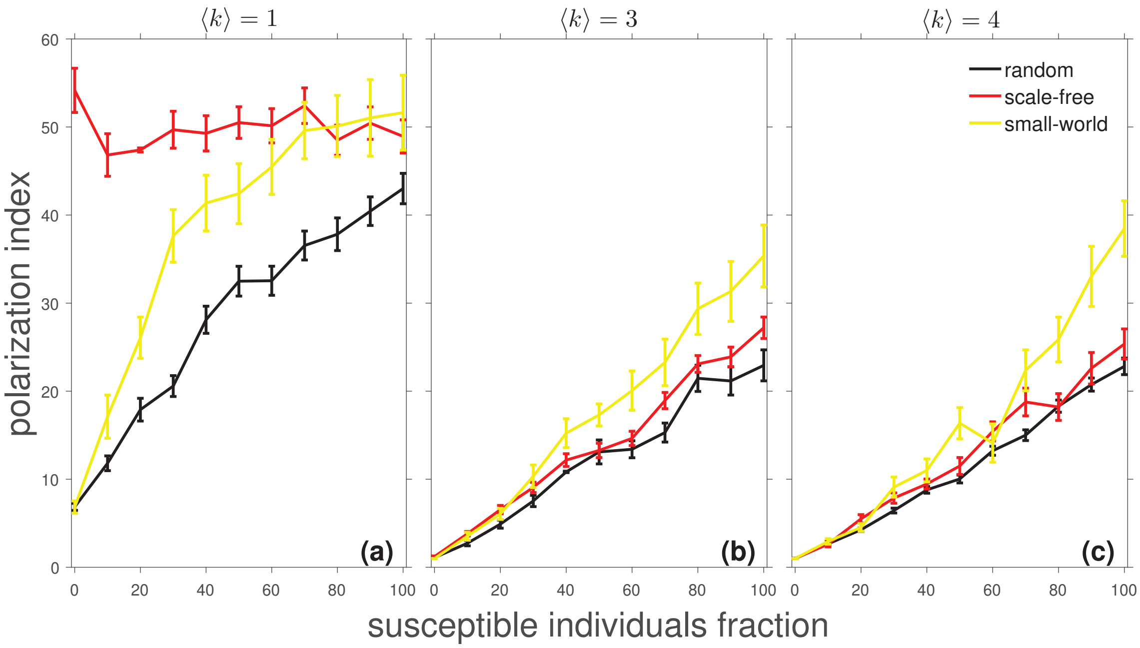

FIGURE 2. Polarization index vs.the susceptible individuals' fraction, when the number of individuals is $N = 200$. The numerical experiments consider average degrees of (a) $\langle k \rangle = 1$, (b) $\langle k \rangle = 3$ and (c) $\langle k \rangle = 4$. Black, red, and yellow lines correspond respectively to random, scale-free, and small-world networks.

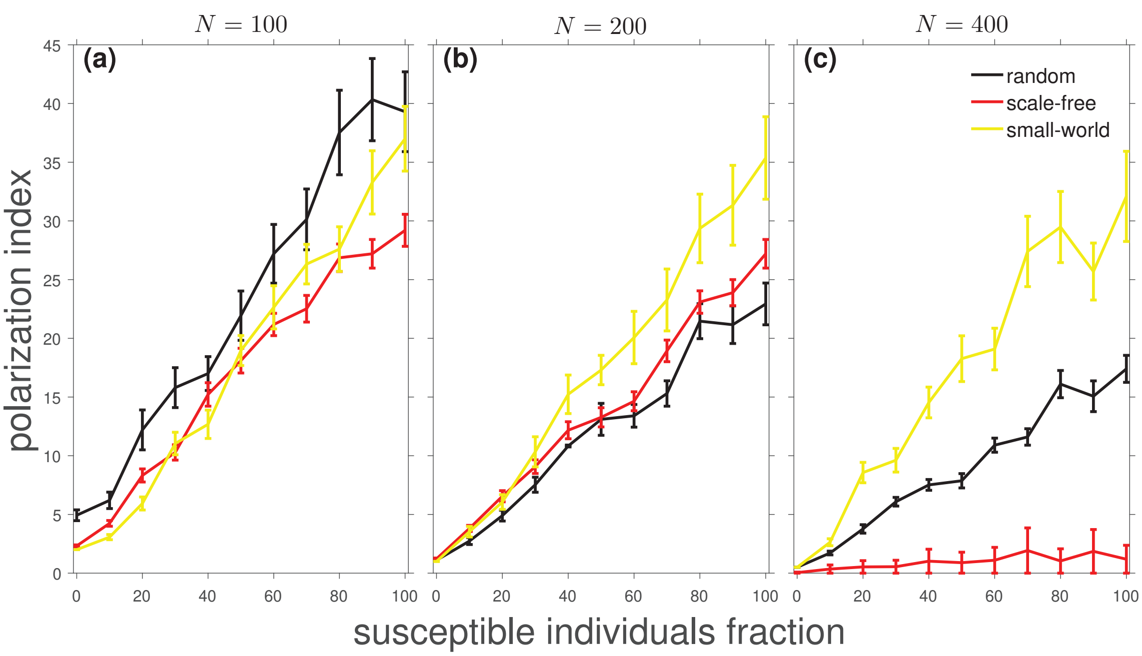

FIGURE 3. Polarization index vs. the susceptible individuals' fraction. The average degree for all the graphics is three. $\langle k \rangle = 3$. The numerical experiments were made for (a) $N = 100$, (b) $N = 200$, and (c) $N = 400$. Black, red, and yellow lines correspond respectively to random, scale-free, and small-world networks.

degree. Finally, our model is complemented by introducing an index that quantifies the tendency of the system to become polarized.

Among the main results, we point out:

- If the susceptible individuals fraction is zero, the polarization also tends to zero; this means that the presence of this type of individuals is essential for the triggering of polarization.
- On the other hand, for all cases, even if there are 100% susceptible individuals, the polarization does not reach 100%.
- For networks with a more significant number of nodes, the polarization decreases for random and scale-free cases, but not in the small-world case.
- The polarization decreases as the average degree increases for the three types of networks.
- For the random and scale-free networks, the polarization ceases to depend on the average degree in huge social groups.
- Small-world networks seem to facilitate the emergence of polarization compared to other networks.

Although we analyzed several features on the emergence of polarization, we pretend to include the role of stabilization times in future works. We also intend to extend our studies to directed and weighted networks.

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